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Strain Energy of Thermally Stressed Multilayer Panels and Its Sensitivity Coefficients

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A study is made of the effect of lamination and material parameters of thermally stressed multilayer composite panels on the strain energy and its sensitivity coefficients obtained by the three-dimensional thermoelasticity model. The sensitivity coefficients measure the sensitivity of the strain energy to variations in the different lamination and material parameters of the panel. The strain energy and sensitivity coefficients obtained by the three-dimensional model are used as the basis for assessing the accuracy of the corresponding quantities obtained by two-dimensional first-order shear deformation theory. Numerical results are presented for antisymmetrically laminated angle-ply panels subjected to linear temperature variation through the thickness. The numerical results demonstrate the accuracy of the strain energy and its sensitivity coefficients obtained by the first-order shear deformation theory, for the range of parameters considered, and the importance of the particular terms in the strain energy, which vanish for single-layer panels.

Nomenclature

$A_{\beta\gamma\rho\delta}$, $B_{\beta\gamma\rho\delta}$, $D_{\beta\gamma\rho\delta}$, $A_{\beta3\rho3}$ = extensional, bending-extensional, bending, and transverse shear coefficients of the panel, respectively
$c_{\beta\gamma\rho\delta}$, $\bar{c}_{\beta\gamma\rho\delta}$ = elastic and reduced stiffnesses of the material, respectively
d_ℓ = lamination and material parameters of the panel
E_L, E_T = elastic moduli of the individual layers in the direction of fibers and normal to it, respectively
G_{LT}, G_{TT} = shear moduli of the individual layers in the plane of fibers and normal to it, respectively
h = total thickness of the panel
L = side length of the panel
$M_{\beta\gamma}$ = bending stress resultants
$N_{\beta\gamma}$ = in-plane (extensional) stress resultants
$\bar{N}_{\beta\gamma}$, $\bar{M}_{\beta\gamma}$ = thermal effects; see Eqs. (6), (A6), and (A9)
NL = total number of layers
Q_β = transverse shear stress resultants
S = surface area of the middle plane of the panel
T = change in the temperature field, measured from a base temperature at which thermal strains are zero
T_0, T_1 = uniform temperature and temperature gradient through the thickness
U, U^c = total strain and complementary energies of the panel, respectively
$\bar{U}$ = terms that distinguish the strain energy of a multilayer panel from that of the corresponding single-layer panel, $\bar{U} = 0$ for single-layer panels
$\hat{U}$ = strain energy density (energy per unit volume) for the panel
U^0 = total strain energy for single-layer panels
x_1, x_2, x_3 = orthogonal coordinate system
α_{ij} = coefficients of thermal expansion
α_L, α_T = coefficients of thermal expansion of the individual layers in the direction of fibers and normal to it, respectively
ϵ_{33} = transverse normal strain component
$\epsilon_{\beta\gamma}$, $\epsilon^0_{\beta\gamma}$ = in-plane strain components at any point in the panel and at the middle plane, respectively
$2\epsilon_{\beta3}$ = transverse shear strain components
θ = fiber orientation angles of the individual layers
$\kappa^0_{\beta\gamma}$ = curvature changes of the panel
ν_{LT}, ν_{TT} = Poisson's ratios of the material of the individual layers
$\sigma_{\beta\gamma}$ = extensional stress components
$\sigma_{\beta3}$ = transverse shear stress components
σ_{33} = transverse normal stress

Subscripts

$\beta, \gamma, \rho, \delta$ $\equiv 1, 2$
i, j, m, n $\equiv 1, 2, 3$
ℓ = 1 to the number of lamination and material parameters
k = kth layer
2d = quantities obtained by first-order shear deformation theory
3d = quantities obtained by three-dimensional thermoelasticity theory

Superscript

k = kth layer

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Introduction

THE unique properties of advanced composite materials, including high specific strength and stiffness, high fatigue resistance, high damping, and near-zero coefficient of thermal expansion in the fiber direction, have resulted in their expanded use in structures subjected to high temperatures. Although the first known application of lamination theory to thermally stressed panels dates back to 1952 (Ref. 1), no studies have been made of the effect of lamination and material parameters on the strain energy in these panels. Since the strain energy density is one of the response characteristics used in identifying damage initiation in composites, an understanding of the effect of variation in the lamination and material parameters of thermally stressed panels on their strain energies is desirable. Also, because of the uncertainty in material properties, there is a need for evaluating the derivatives of the strain energy of multilayered panels with respect to the various material and lamination parameters.

In this paper a study is made of the effects of lamination and material parameters of thermally stressed multilayer panels on the total strain energy and its sensitivity coefficients obtained by the three-dimensional thermoelasticity model. The sensitivity coefficients refer to the derivatives of the strain energy with respect to the different lamination and material parameters of the panel. The strain energy and sensitivity coefficients obtained by the three-dimensional model are used as the basis for assessing the accuracy of the corresponding quantities obtained by two-dimensional first-order shear deformation theory. Numerical results are presented showing the accuracy of the total strain energy obtained by using the first-order shear deformation model and the importance of the particular terms in the strain energy, which vanish for single-layer panels. The standard of comparison is taken to be the strain energy obtained by using the three-dimensional thermoelasticity model.

The laminates considered herein are flat and consist of an arbitrary number of perfectly bonded layers. The individual layers are assumed to be initially homogeneous and anisotropic. At each point of the laminate, a plane of thermoelastic symmetry exists parallel to the middle plane of the panel. The sensitivity coefficients developed in the present study can be used 1) to assess the effects of uncertainties, in the material parameters of the panel, on the strain energy; 2) to predict the changes in the strain energy due to changes in the material parameters of the laminate; and 3) to identify the changes in material parameters required to improve certain performance characteristics of the panel.

Expressions for Strain and Complementary Energies and Their Sensitivity Coefficients

Total Strain and Complementary Energies

If a three-dimensional thermoelasticity model is used, the expressions for the total strain and complementary energies for a thermally stressed laminated panel made of a thermoelastic material, with one plane of thermoelastic symmetry parallel to the middle plane, can be written in the following form:

$$U = \frac{1}{2}\int_S \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} [\sigma_{\beta\gamma}(\epsilon_{\beta\gamma} - \alpha_{\beta\gamma}T) + 2\sigma_{\beta 3}\epsilon_{\beta 3} + \sigma_{33}(\epsilon_{33} - \alpha_{33}T)]^{(k)}\, dx_3\, dS \tag{1}$$

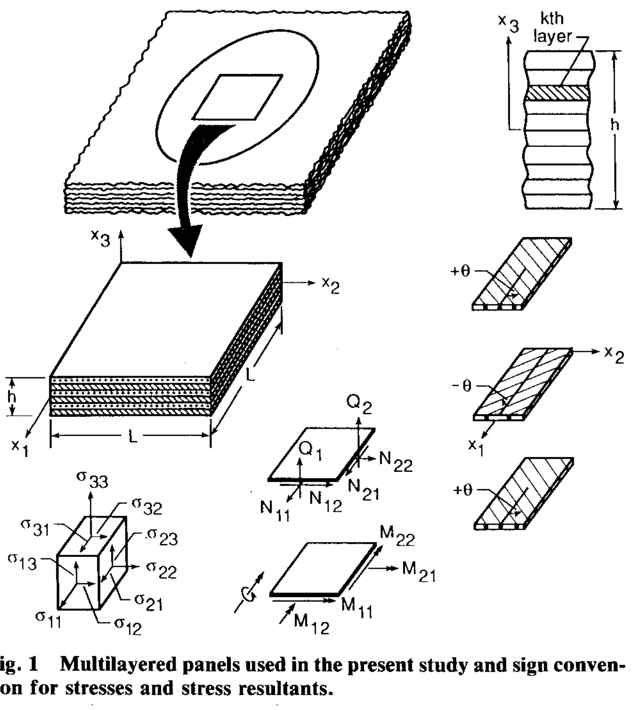


Fig. 1 Multilayered panels used in the present study and sign convention for stresses and stress resultants.

$$U^c = U + \int_S \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} [(\sigma_{\beta\gamma}\alpha_{\beta\gamma} + \sigma_{33}\alpha_{33})T]^{(k)}\, dx_3\, dS \tag{2}$$

In Eqs. (1) and (2) the quantities between square brackets are the strain energy density; $U_c - U$ is the difference between the complementary energy density and the strain energy density; S is the surface area of the panel; k is the layer number; the range of the Greek indices β, γ is 1, 2; and a repeated index in the same term denotes summation over its full range. Note that Eqs. (1) and (2) are extensions of the corresponding expressions for isotropic materials presented in Ref. 2.

If a first-order shear deformation theory is used in modeling the panel, and the constitutive equations of the Appendix are used, Eqs. (1) and (2) can be written in the following form:

$$U = U^0 + \bar{U} \tag{3}$$

and

$$U^c = U + \int_S \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} [\sigma_{\beta\gamma}\alpha_{\beta\gamma}T]^{(k)}\, dx_3\, dS \tag{4}$$

where

$$U^0 = \frac{1}{2}\int_S [N_{\beta\gamma} M_{\beta\gamma} Q_\beta] \begin{bmatrix} A_{\beta\gamma\rho\delta} & B_{\beta\gamma\rho\delta} & \cdot \\ B_{\beta\gamma\rho\delta} & D_{\beta\gamma\rho\delta} & \cdot \\ \cdot & \cdot & A_{\beta 3\rho 3} \end{bmatrix}^{-1} \begin{bmatrix} N_{\rho\delta} \\ M_{\rho\delta} \\ Q_\rho \end{bmatrix} dS \tag{5}$$

$$\bar{U} = \frac{1}{2}\int_S [N_{\beta\gamma} M_{\beta\gamma}] \begin{bmatrix} A_{\beta\gamma\rho\delta} & B_{\beta\gamma\rho\delta} \\ B_{\beta\gamma\rho\delta} & D_{\beta\gamma\rho\delta} \end{bmatrix}^{-1} \begin{bmatrix} \bar{N}_{\rho\delta} \\ \bar{M}_{\rho\delta} \end{bmatrix} dS - \frac{1}{2}\int_S \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} [\sigma_{\beta\gamma}\alpha_{\beta\gamma}T]^{(k)}\, dx_3\, dS \tag{6}$$

The range of the Greek indices is 1, 2, and repeated indices in Eqs. (4–6) denote summation over their full range. The sign convention for the stresses and stress resultants is given in Fig. 1. As can be seen from Eqs. (5) and (6), the quantity U^0 is evaluated in terms of the stress resultants and the effective stiffnesses (or compliances) of the panel. By contrast, the second term of $\bar{U}$ has to be evaluated layer by layer.

Note that in the first-order shear deformation theory the total strain components $\epsilon_{\beta\gamma}$ are assumed to have a linear variation through the thickness [see Eq. (A2)]. However, because of the possible discontinuity of the thermal strains $\alpha_{\beta\gamma}T$ in the thickness direction of the laminate, the mechanical strains can be discontinuous. This is depicted in Fig. 2 in which the thickness variation of the total and mechanical strain components ϵ_{11} and ϵ_{22}, obtained by three-dimensional thermoelasticity theory, is shown for a 10-layer cross-ply laminate subjected to linear temperature variation through the thickness $T = T_0 + x_3 T_1$. In the absence of thermal strains, the mechanical strains are continuous through the thickness, $\bar{U}$ vanishes, and the total strain energy equals U^0.

The quantity $\bar{U}$ in Eq. (3) distinguishes the strain energy of a general multilayer panel from that of the corresponding single-layer panel (or multilayer panel with continuous thermal strain distribution through the thickness, for example, a panel having the same fiber orientation in the different layers). Note that $\bar{U}$ vanishes when T has a linear variation, and the mechanical strains are continuous, through the thickness, in which case the total strain energy is equal to U^0. The corresponding expression for the total strain energy U based on the classical lamination theory is given in Ref. 3.

Sensitivity Coefficients

The derivatives of the total strain and total complementary energies, with respect to the lamination and material parameters d_ℓ, are obtained by differentiating Eqs. (3–6). The resulting expressions for the first-order shear deformation model are

$$\frac{\partial U}{\partial d_\ell} = \frac{\partial U^0}{\partial d_\ell} + \frac{\partial \bar{U}}{\partial d_\ell} \tag{7}$$

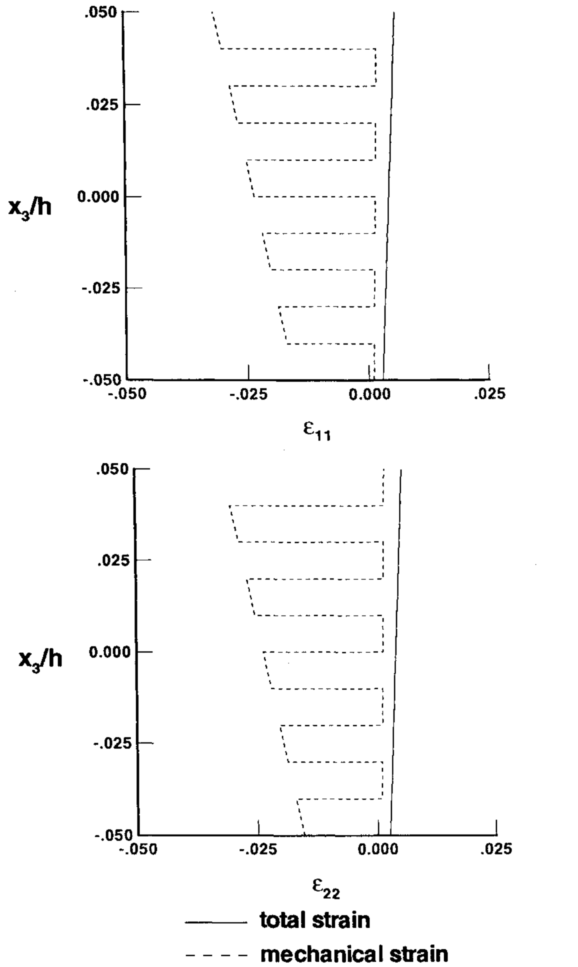


Fig. 2 Variation in total and mechanical strain components through the thickness of a square, 10-layer, cross-ply laminate subjected to a linear temperature variation through the thickness, $T = T_0 + x_3T_1$ in the thickness direction, $h/L = 0.02$, fiber orientation [0/90]5, $h\ T_1/T_0 = 0.7$.

$$\frac{\partial U^c}{\partial d_\ell} = \frac{\partial U}{\partial d_\ell} + \int_S \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} \left[\left(\frac{\partial \sigma_{\beta\gamma}}{\partial d_\ell} \alpha_{\beta\gamma} + \sigma_{\beta\gamma} \frac{\partial \alpha_{\beta\gamma}}{\partial d_\ell} \right) T \right]^{(k)} \mathrm{d}x_3\, \mathrm{d}S \tag{8}$$

where

$$\frac{\partial U^0}{\partial d_\ell} = \frac{1}{2} \int_S \left(2[N_{\beta\gamma}\, M_{\beta\gamma}\, Q_\beta] \begin{bmatrix} A_{\beta\gamma\rho\delta} & B_{\beta\gamma\rho\delta} & \cdot \\ B_{\beta\gamma\rho\delta} & D_{\beta\gamma\rho\delta} & \cdot \\ \cdot & \cdot & A_{\beta 3\rho 3} \end{bmatrix}^{-1} \frac{\partial}{\partial d_\ell} \begin{Bmatrix} N_{\rho\delta} \\ M_{\rho\delta} \\ Q_\rho \end{Bmatrix} \right.$$

$$\left. + [N_{\beta\gamma}\, M_{\beta\gamma}\, Q_\beta] \frac{\partial}{\partial d_\ell} \begin{bmatrix} A_{\beta\gamma\rho\delta} & B_{\beta\gamma\rho\delta} & \cdot \\ B_{\beta\gamma\rho\delta} & D_{\beta\gamma\rho\delta} & \cdot \\ \cdot & \cdot & A_{\beta 3\rho 3} \end{bmatrix}^{-1} \begin{Bmatrix} N_{\rho\delta} \\ M_{\rho\delta} \\ Q_\rho \end{Bmatrix} \right) \mathrm{d}S \tag{9}$$

and

$$\frac{\partial \bar{U}}{\partial d_\ell} = \frac{1}{2} \int_S \left(\frac{\partial}{\partial d_\ell} [N_{\beta\gamma}\, M_{\beta\gamma}] \begin{bmatrix} A_{\beta\gamma\rho\delta} & B_{\beta\gamma\rho\delta} \\ B_{\beta\gamma\rho\delta} & D_{\beta\gamma\rho\delta} \end{bmatrix}^{-1} \begin{Bmatrix} \bar{N}_{\rho\delta} \\ \bar{M}_{\rho\delta} \end{Bmatrix} \right.$$

$$+ [N_{\beta\gamma}\, M_{\beta\gamma}] \frac{\partial}{\partial d_\ell} \begin{bmatrix} A_{\beta\gamma\rho\delta} & B_{\beta\gamma\rho\delta} \\ B_{\beta\gamma\rho\delta} & D_{\beta\gamma\rho\delta} \end{bmatrix}^{-1} \begin{Bmatrix} \bar{N}_{\rho\delta} \\ \bar{M}_{\rho\delta} \end{Bmatrix}$$

$$+ [N_{\beta\gamma}\, M_{\beta\gamma}] \begin{bmatrix} A_{\beta\gamma\rho\delta} & B_{\beta\gamma\rho\delta} \\ B_{\beta\gamma\rho\delta} & D_{\beta\gamma\rho\delta} \end{bmatrix}^{-1} \frac{\partial}{\partial d_\ell} \begin{Bmatrix} \bar{N}_{\rho\delta} \\ \bar{M}_{\rho\delta} \end{Bmatrix}$$

$$\left. - \frac{1}{2} \int_S \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} \left[\left(\frac{\partial \sigma_{\beta\gamma}}{\partial d_\ell} \alpha_{\beta\gamma} + \sigma_{\beta\gamma} \frac{\partial \alpha_{\beta\gamma}}{\partial d_\ell} \right) T \right]^{(k)} \mathrm{d}x_3 \right) \mathrm{d}S \tag{10}$$

Numerical Studies

Numerical studies have been conducted to provide some insight into the effects of variations in the lamination and material parameters of composite panels on 1) the strain energy and its sensitivity coefficients obtained by the three-dimensional thermoelasticity theory, 2) the accuracy of the total strain energy and its sensitivity coefficients predicted by the first-order shear deformation theory, and 3) the importance of the quantity $\bar{U}$, which distinguishes the strain energy of a general multilayer panel from that of the corresponding single-layer panel. For each problem considered the total strain energy and its sensitivity coefficients (derivatives of the strain energy with respect to various material and lamination parameters) obtained by the first-order shear deformation theory were compared with those obtained by the three-dimensional thermoelasticity theory. A typical problem set is considered herein, namely that of the linear thermoelastic static response of square angle-ply laminates with $L = 1$. The laminates have antisymmetric lamination with respect to the middle plane with fiber orientation alternating between $+\theta$ and $-\theta$. The panels are subjected to either a uniform temperature through the thickness, $T = T_0 \cos(\pi x_1/L) \cos(\pi x_2/L)$, or a uniform temperature gradient through the thickness $T = x_3 T_1 \sin(\pi x_1/L) \sin(\pi x_2/L)$ (where T_0 and T_1 are constants). Henceforth, these two cases will be referred to as the T_0 case and T_1 case, respectively. The thermoelastic response is assumed to be periodic in both x_1 and x_2 directions with periods equal to $2L$. The material properties of the individual layers are taken to be those typical of high-modulus fibrous composites, namely, $E_L/E_T = 15$, $G_{LT}/E_T = 0.52$, $G_{TT}/E_T = 0.3378$, $\nu_{LT} = 0.32$, $\nu_{TT} = 0.48$, $\alpha_L/\alpha_T = 10^{-10}$, $\alpha_T T_0 = 1$, and $\alpha_T T_1 = 1$, where the subscript L refers to the direction of fibers, and the subscript T refers to the transverse direction.

Exact three-dimensional solutions are obtained for this problem, using the procedure described in Refs. 4 and 5, and are used as the standard for comparison. The solutions ob-

a)

b)

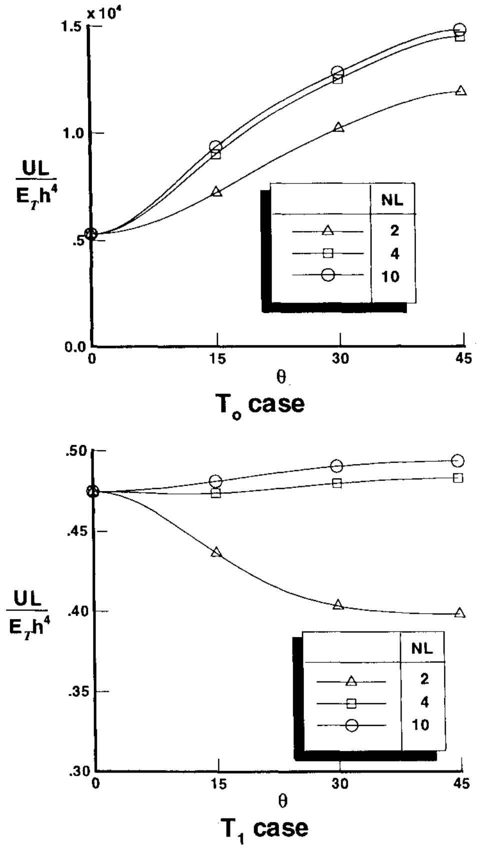


Fig. 3 Effect of fiber orientation θ and number of layers NL on the total strain energy obtained by the three-dimensional thermoelasticity model.

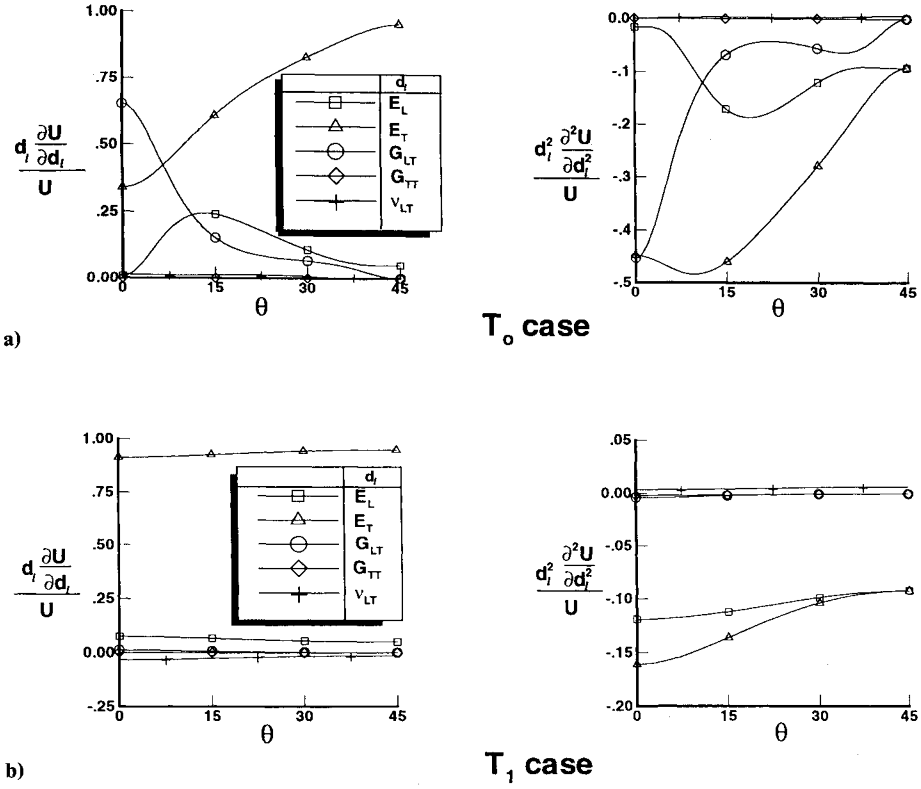


Fig. 4 Effect of fiber orientation θ on the sensitivity coefficients of the strain energy obtained by the three-dimensional thermoelasticity model; NL = 10.

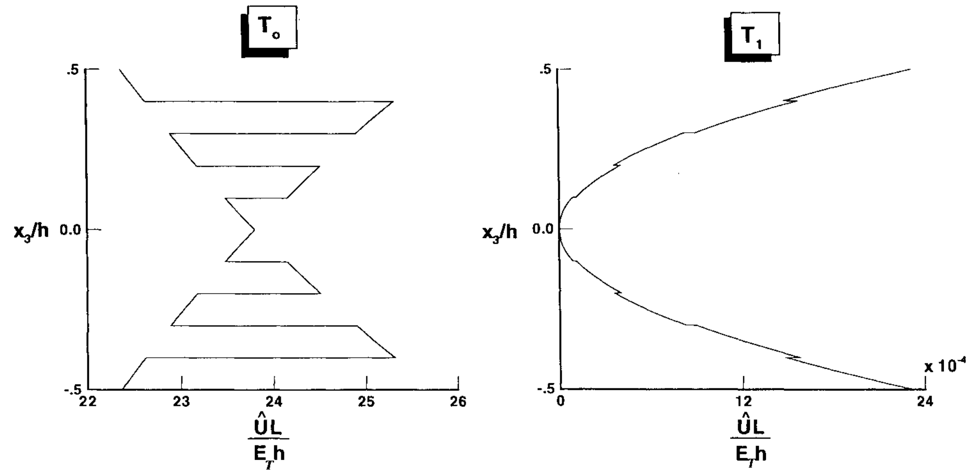


Fig. 5 Distribution of the strain energy density in the thickness direction; NL = 10 and θ = 45 deg.

tained using the first-order shear deformation theory are also exact and correspond to a value of 5/6 for the composite shear correction factors. The plates considered cover a wide range of geometric and lamination parameters: thickness ratio h/L varying between 0.01 and 0.2, number of layers NL ranging from 2 to 10, and fiber orientation angle θ ranging from 0 to 45 deg. Typical results are shown in Figs. 3–8 for square, antisymmetrically laminated, angle-ply panels with $h/L = 0.02$ and are discussed subsequently.

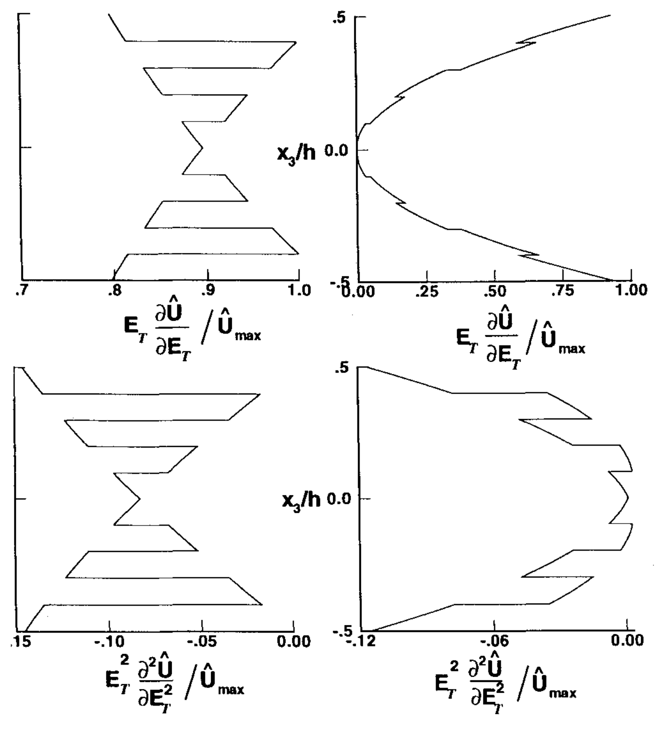
The effects of the fiber orientation θ and the number of layers NL on the total strain energy U obtained by the three-dimensional thermoelasticity theory are shown in Fig. 3. Whereas for the T_0 case, U increases with the increase in both θ and NL, for the T_1 case, U decreases with the increase in θ when $NL = 2$. However, for $NL \geq 4$, U increases with the increase in θ. The decrease in U for $NL = 2$, generated by T_1, is attributed to the large bending-extensional coupling effects.

The variations of the first- and second-order sensitivity coefficients of U with θ are shown in Fig. 4 for 10-layer panels. The sensitivity coefficients are each normalized through multiplying by λ (or λ^2) and dividing by U. For the T_0 case, U is very sensitive to variations in E_T, somewhat sensitive to variations in E_L (for $\theta \geq 10$ deg) and G_{LT} (particularly for $\theta \leq 10$ deg), and insensitive to variations in G_{TT} and ν_{LT}. For the T_1 case, U is very sensitive to variations in E_T, somewhat sensitive to variations in E_L, less sensitive to variations in ν_{LT}, and insensitive to variations in G_{LT} and G_{TT}. For $\theta \geq 30$ deg, the normalized second-order sensitivity coefficients with respect to both E_L and E_T are of the same magnitude (in the T_1 case).

For both the T_0 and T_1 cases, the normalized sensitivity coefficients with respect to the thermal coefficients of expansion α_L and α_T do not vary with θ (results not shown in Fig. 4). The strain energy U is very sensitive to variations in α_T but is insensitive to variations in α_L.

In general, the normalized first-order and second-order sensitivity coefficients, obtained by the three-dimensional thermoelasticity model, are insensitive to variations in h/L. Exceptions to this are the second-order sensitivity coefficients with respect to E_T, G_{LT}, and G_{TT} for the T_1 case. Each of these coefficients increases with increasing h/L.

The distributions of the strain energy density $\hat{U}$ and its first-order and second-order sensitivity coefficients with respect to E_T in the thickness direction are shown in Figs. 5 and 6 for a 10-layer angle-ply panel with $\theta = 45$ deg. Note that for the T_0 case $\hat{U}$ and its sensitivity coefficients are piecewise linear, and for the T_1 case they are piecewise nonlinear.



a) b)

Fig. 6 Distribution of the sensitivity coefficients of the strain energy density in the thickness direction; NL = 10 and θ = 45 deg.

An indication of the accuracy of the total strain energy U obtained by the first-order shear deformation theory for different values of θ and NL is shown in Fig. 7 for panels with $h/L = 0.02$. Also, the ratio of U^0 [Eq. (5)] to the total strain energy obtained by the three-dimensional thermoelastic model U_{3d} is shown in Fig. 7. As can be seen from Fig. 7, U obtained by using Eqs. (3), (5), and (6) is very accurate for all of the panels considered. On the other hand, U^0 can grossly underestimate the total strain energy of the panel. The difference between U^0 and U [$\bar{U}$ in Eqs. (3) and (6)] increases rapidly with increasing θ (from 0 to 45 deg) and increasing NL (from 2 to 4) and increases slowly with increasing NL beyond 4.

As is to be expected, the error in U obtained by the first-order shear deformation theory increases with the increase in h/L. For two-layer panels with $\theta = 30$ deg and $h/L = 0.2$, the maximum errors in U for the T_0 and T_1 cases are 10.9 and

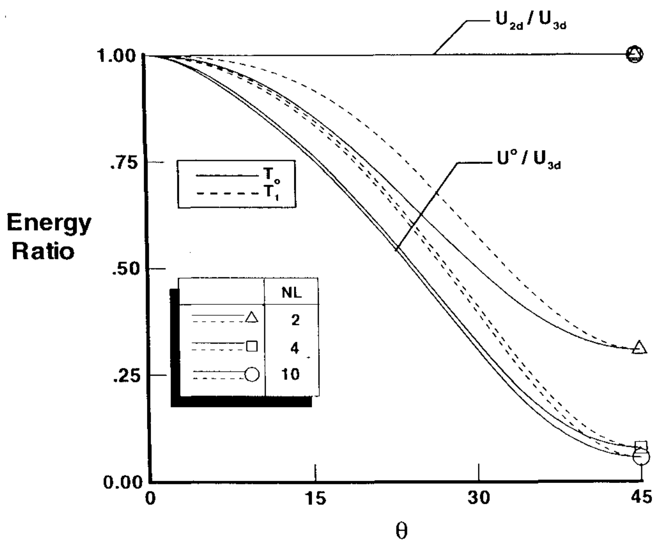


Fig. 7 Effect of fiber orientation θ and number of layers NL on the energy ratios, U_{2d}/U_{3d} and U^0/U_{3d}; NL = 10. Solid lines are for uniform temperature, and dashed lines are for uniform temperature gradient through the thickness.

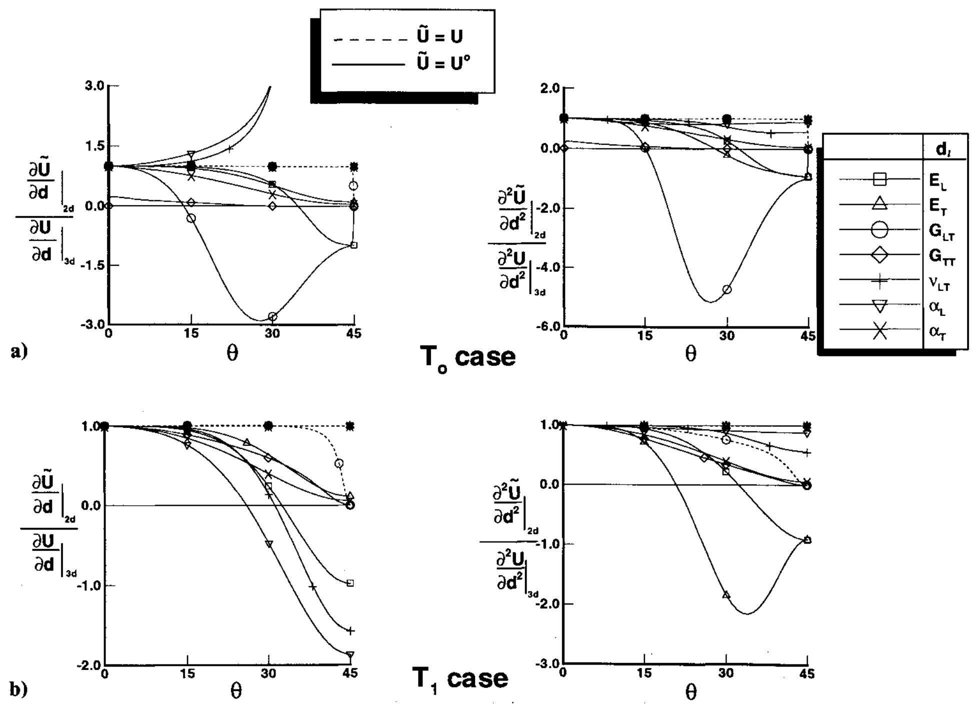


Fig. 8 Effect of fiber orientation θ on the ratio of the sensitivity coefficients obtained by the first-order shear deformation theory to the corresponding ones obtained by three-dimensional thermoelasticity theory; NL = 10.

3.5%, respectively. The corresponding errors for the 10-layer panels are less than 0.5%.

An indication of the accuracy of the first- and second-order sensitivity coefficients of the total strain energy, obtained by the first-order shear deformation theory, for different values of θ is shown in Fig. 8. Also shown are the ratios of the sensitivity coefficients of U^0 to the sensitivity coefficients of U, obtained by the three-dimensional thermoelasticity model. Note that, because of the absence of thermal transverse shear strains in the laminates considered herein, the derivatives of U and U^0 with respect to G_{TT} are equal. The corresponding derivatives of U and U^0 with respect to all other material parameters are not equal.

As can be seen from Fig. 8, the sensitivity coefficients of U obtained by the first-order shear deformation theory are fairly accurate. Exceptions to this are the small first- and second-order sensitivity coefficients with respect to G_{TT} for the T_0 and T_1 cases, the small first- and second-order sensitivity coefficients with respect to G_{LT} for $40 \le \theta \le 45$ deg, and the small second-order sensitivity coefficients with respect to G_{LT} for the T_1 case (see dashed lines in Fig. 8). The sensitivity coefficients of U^0 can be considerably different from the sensitivity coefficients of U, and the contribution of $\bar{U}$ to the sensitivity coefficients [Eqs. (7) and (10)] must be included in the sensitivity calculations. This is particularly true for $\theta \ge 10$ deg.

The errors in the sensitivity coefficients obtained by the first-order shear deformation theory increase only slightly with the increase in h/L. For 10-layer panels with $\theta = 30$ deg and $h/L = 0.2$, the maximum errors in the large first-order and second-order sensitivity coefficients are less than 4 and 7% for the T_0 and T_1 cases, respectively.

Concluding Remarks

A study is made of the effect of lamination and material parameters of thermally stressed multilayer composite panels on the strain energy and its sensitivity coefficients obtained by the three-dimensional thermoelasticity model. An assessment is made of the accuracy of the strain energy and sensitivity coefficients obtained by the two-dimensional first-order shear deformation model. The composite shear correction factors were selected to be 5/6. Numerical results are presented for antisymmetrically laminated angle-ply panels subjected to linear temperature variation through the thickness. The numerical studies presented show that, for the case of uniform temperature through the thickness, the total strain energy is very sensitive to variations in E_T, somewhat sensitive to variations in E_L and G_{LT}, and insensitive to variations in G_{TT} and ν_{TT}. For the case of uniform temperature gradient through the thickness, the total strain energy is very sensitive to variations in E_T, somewhat sensitive to variations in E_L, less sensitive to variations in ν_{LT}, and insensitive to variations in G_{LT} and G_{TT}. The numerical studies presented also show that, for the range of the parameters considered, the total strain energy and its large sensitivity coefficients obtained by the first-order shear deformation theory are fairly accurate and that gross errors in both the strain energy and its sensitivity coefficients can result from neglecting the terms that distinguish multilayer panels from the corresponding single-layer panels. For panels with $h/L \le 0.2$, the errors in the predictions of the first-order shear deformation theory increase slightly with the increase in the thickness ratio of the panel. For thick panels with $h/L \ge 0.2$, accurate strain energy and sensitivity coefficients can be obtained by using the predictor-corrector approaches described in Ref. 7.

Appendix: Thermoelastic Constitutive Relations for Multilayered Composite Panels

For a linear three-dimensional thermoelastic solid, the Duhammel-Neumann type constitutive model can be expressed by the following relations (Refs. 3 and 6):

$$\sigma_{ij} = c_{ijmn}(\epsilon_{mn} - \alpha_{mn}T) \tag{A1}$$

where σ_{ij} and ϵ_{mn} are the stress and strain components; c_{ijmn} are the elastic stiffnesses of the material; α_{mn} are the coefficients of thermal expansion of the material; T is the change in the temperature field, measured from a base (reference) temperature at which the thermal strains are zero; $i, j, m, n = 1$–3; and repeated indices m and n denote summation over their full range.

A first-order shear deformation theory is based on the following assumptions: 1) linear through-the-thickness distribution of the in-plane displacement and extensional strain components, 2) negligible transverse normal strain component ϵ_{33}, and 3) negligible effect of the transverse stress component σ_{33} (in the constitutive relations). These assumptions can be expressed by the following relations for the strain and stress components:

$$\epsilon_{\beta\gamma} = \epsilon^0_{\beta\gamma} + x_3\kappa^0_{\beta\gamma} \tag{A2}$$

$$2\epsilon_{\beta 3} = 2\epsilon^0_{\beta 3} \tag{A3}$$

$$\epsilon_{33} = 0 \tag{A4}$$

and

$$\sigma_{\beta\gamma} = \bar{c}_{\beta\gamma\rho\delta}(\epsilon^0_{\rho\delta} + x_3\kappa^0_{\rho\delta} - \alpha_{\rho\delta}T) \tag{A5}$$

where $\epsilon^0_{\rho\delta}$ and $\kappa^0_{\rho\delta}$ are the extensional and bending strains of the middle surface of the panel; $2\epsilon^0_{\beta 3}$ are the average transverse shear strains of the panel; $\bar{c}_{\beta\gamma\rho\delta}$ are the reduced stiffnesses of the material; $\beta, \gamma, \rho, \delta = 1, 2$; and repeated indices ρ and δ denote summation over their full range.

The constitutive relations for the panel can be written in the following form:

$$\begin{Bmatrix} N_{\beta\gamma} \\ M_{\beta\gamma} \\ Q_\beta \end{Bmatrix} = \begin{bmatrix} A_{\beta\gamma\rho\delta} & B_{\beta\gamma\rho\delta} & \cdot \\ B_{\beta\gamma\rho\delta} & D_{\beta\gamma\rho\delta} & \cdot \\ \cdot & \cdot & A_{\beta 3\rho 3} \end{bmatrix} \begin{Bmatrix} \epsilon^0_{\rho\delta} \\ \kappa^0_{\rho\delta} \\ 2\epsilon^0_{\rho 3} \end{Bmatrix} - \begin{Bmatrix} \bar{N}_{\beta\gamma} \\ \bar{M}_{\beta\gamma} \\ \cdot \end{Bmatrix} \tag{A6}$$

where $N_{\beta\gamma}$, $M_{\beta\gamma}$, and Q_β are the extensional, bending, and transverse shear stress resultants; the various A and B are panel stiffness coefficients; $\bar{N}_{\beta\gamma}$ and $\bar{M}_{\beta\gamma}$ represent thermal effects; a dot (·) refers to a zero component; and repeated indices ρ and δ in the same term denote summation over the range 1 and 2. Note that in Eqs. (A6) the panel is assumed to have, at each point, a plane of thermoelastic symmetry parallel to its middle plane. The expressions for the various A, B, D, $\bar{N}$, and $\bar{M}$ are given by

$$\begin{Bmatrix} A_{\beta\gamma\rho\delta} \\ B_{\beta\gamma\rho\delta} \\ D_{\beta\gamma\rho\delta} \end{Bmatrix} = \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} c_{\beta\gamma\rho\delta} \begin{Bmatrix} 1 \\ x_3 \\ (x_3)^2 \end{Bmatrix} \mathrm{d}x_3 \tag{A7}$$

$$A_{\beta 3\rho 3} = \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} c_{\beta 3\rho 3}\, \mathrm{d}x_3 \tag{A8}$$

$$\begin{Bmatrix} \bar{N}_{\beta\gamma} \\ \bar{M}_{\beta\gamma} \end{Bmatrix} = \sum_{k=1}^{NL} \int_{h_{k-1}}^{h_k} \bar{c}_{\beta\gamma\rho\delta}\alpha_{\rho\delta}T \begin{Bmatrix} 1 \\ x_3 \end{Bmatrix} \mathrm{d}x_3 \tag{A9}$$

where k refers to the layer number.

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